A-T Volume Integral Formulation for Solving Electromagnetic Problems in the frequency domain

Gérard Meunier, Olivier Chadebec, Jean-Michel Guichon, Vinh Le-Van

Univ. Grenoble Alpes, G2Elab, F-38000 Grenoble, France
CNRS, G2Elab, F-38000 Grenoble, France
gerard.meunier@g2elab.grenoble-inp.fr

***Abstract* — Volume integral formulations for solving electromagnetic problems in the frequency domain are proposed. Firstly, it is based on a magnetic flux B and current density J facet interpolation for representing the electromagnetic problem through an equivalent circuit. Secondly, magnetic vector potentials A and electric vector potential T are considered thanks to discrete geometric approach. The formulations are particularly well adapted for solving electromagnetic problems with large air domains.**

***Index Terms* — volume integral formulation, equivalent circuit, facet and edge elements, vector potential, electromagnetism.**

I. Introduction

Different works have shown the interest of using Volume Integral Method (VIM) for 3D magnetic field analysis [1] the main advantage being that the air region does not need to be meshed. On the other hand, Whitney facet interpolation for current density **J** and magnetic flux density **B** is well suited for representing an electromagnetic problem through an equivalent circuit [2]. This approach provides a solution [3] that ensures the conservation of flux and current through the facets of the mesh. In this work we propose an alternative vector potential approach which is derived from the previous method thanks to the use of discrete geometric approach.

II. Equivalent Circuit Approach Through Facet Element And Volume Integral Formulation

Let us consider a problem with volume magnetic regions Ω_m (presence of magnetization), volume conductive electrical regions Ω_c (presence of current density) and coils (imposed current density $\mathbf{J}_0$). The two laws linking respectively the current density **J** to the electric field **E** (in Ω_c) and the flux density **B** to the magnetic field **H** (in Ω_m) are expressed as follow:

$$\mathbf{J} = \sigma\,\mathbf{E} \qquad \mathbf{H} = \nu\,\mathbf{B} \tag{1}$$

In [3], the authors propose to solve electromagnetic problems through a volume integral formulation based on Withney first order facet finite elements discretization for **B** and **J** (the mesh is limited to Ω_m for **B** and to Ω_c for **J**) :

$$\mathbf{J} = \sum_j \mathbf{w}_j\, I_j \qquad \mathbf{B} = \sum_g \mathbf{w}_g\, \Psi_g \tag{2}$$

where $\mathbf{w}_j$ and $\mathbf{w}_g$ are facet shape functions and I_j, Ψ_g the fluxes across the facets. The problem can then be represented by two equivalent circuits (see Fig. 1), dual meshes of the primal meshes used for **B** (Ω_m) and **J** (Ω_c). For a low frequency problem we obtain the circuit equation system:

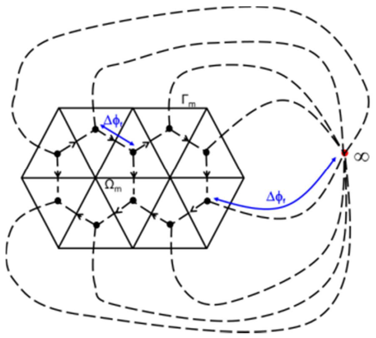

Fig.1 Circuit representation for magnetic regions. A similar representation is obtained for electric regions (with ΔV and I)

$$\begin{bmatrix} Z & Y \\ F & K \end{bmatrix} \begin{Bmatrix} I \\ \Psi \end{Bmatrix} = \begin{Bmatrix} \Delta V \\ \Delta\Phi \end{Bmatrix} + \begin{Bmatrix} U \\ Q \end{Bmatrix} \tag{3}$$

with

$$\begin{aligned}
Z_{ij} &= \int_{\Omega_c} \mathbf{w}_i . \frac{\mathbf{w}_j}{\sigma} d\Omega + j\omega \frac{\mu_0}{4\pi} \int_{\Omega_c} \mathbf{w}_i . \int_{\Omega_c} \frac{\mathbf{w}_j}{r} d\Omega\, d\Omega \\
Y_{ig} &= j\omega \frac{\mu_0}{4\pi} \int_{\Omega_c} \mathbf{w}_i . \int_{\Omega_m} (\nu_0 - \nu) \frac{\mathbf{w}_g \wedge \mathbf{r}}{r^3} d\Omega\, d\Omega \\
K_{fg} &= \int_{\Omega_m} \mathbf{w}_f\ \nu\, \mathbf{w}_g\, d\Omega + \frac{1}{4\pi} \int_{\Gamma_f} \frac{1}{S_f} \int_{\Omega_m} (\nu - \nu_0) \frac{\mathbf{w}_g . \mathbf{r}}{r^3} d\Omega\, d\Omega \\
F_{fj} &= -\frac{1}{4\pi} \int_{\Omega_c} \mathbf{w}_f . \int_{\Omega_m} \frac{\mathbf{w}_j \wedge \mathbf{r}}{r^3} d\Omega\, d\Omega
\end{aligned} \tag{4}$$

where ω is the angular frequency and r is the distance between integration points in the cases of double integration with the Green kernel. Source terms produced by coils are expressed such as:

$$U_i = -\frac{\mu_0}{4\pi} \int_{\Omega_c} \mathbf{w}_i . \left(\int_{\Omega_0} \frac{\mathbf{J}_0}{r} d\Omega \right) d\Omega \qquad Q_f = \frac{1}{4\pi} \int_{\Omega_m} \mathbf{w}_f . \left(\int_{\Omega_0} \frac{\mathbf{J}_0 \wedge \mathbf{r}}{r^3} d\Omega \right) d\Omega$$

The system (3) can be solved thanks to the use of a circuit solver such as mesh current method. Circuit equations M_m $\{\Delta\Phi\}=0$ and M_c $\{\Delta V\}=0$ [3], where M_c and M_m are the branch-fundamental independent loop matrices, strongly imposes the solenoidality of the magnetic flux **B** and current density **J**. In the next section we propose to solve equations (3) through an alternative vector potentials **A-T** formulation thanks to discrete geometric approach [4] [5].

III. A-T Discrete Geometric Approach

Let us consider usual connectivity matrices of a primal mesh: G between edges and nodes, C between faces and edges, and D between volumes and faces. Matrices of the dual complex are then defined by $\tilde{G}, \tilde{C}, \tilde{D}$ with $\tilde{G} = D^t$, $\tilde{C} = C^t$, $\tilde{D} = -G^t$ [4],[5].

On the primal meshes on which **B** and **J** are interpolated, the connectivity matrices C_m and C_c link the integrals of magnetic and electric vector potential along the edges (denoted A and T), with Ψ and I, the magnetic fluxes and the currents through the facets:

$$\{\Psi\} = C_m \{A\} \quad (\Psi_{facet} = \iint \mathbf{B}\, dS = \oint \mathbf{A}\, dl = \sum_{egdes\ e} A_e)$$
$$\{I\} = C_c \{T\} \quad (I_{facet} = \iint \mathbf{J}\, dS = \oint \mathbf{T}\, dl = \sum_{egdes\ e} T_e) \tag{5}$$

On the dual meshes, which define the equivalent circuits, the connectivity gradient matrices $\tilde{G}_m$ and $\tilde{G}_c$ link edges and nodes such as:

$$\{\Delta\Phi\} = \tilde{G}_m \{\Phi\} \qquad \{\Delta V\} = \tilde{G}_c \{V\} \tag{6}$$

By considering that $\tilde{G} = D^t$ and D.C = 0 [4], we have:

$$C_m^t \{\Delta\Phi\} = C_m^t D_m^t \{\Phi\} = 0 \quad C_c^t \{\Delta V\} = C_c^t D_c^t \{V\} = 0 \tag{7}$$

From (3) and (7), we then obtain the system to be solved depending of the unknowns A and T:

$$\begin{bmatrix} C_c^t\, Z\, C_c & C_c^t\, Y\, C_m \\ C_m^t\, F\, C_c & C_m^t\, K\, C_m \end{bmatrix} \begin{Bmatrix} T \\ A \end{Bmatrix} = \begin{Bmatrix} C_c^t\, U \\ C_m^t\, Q \end{Bmatrix} \tag{8}$$

IV. Application And Discussion

We have tested this **A-T** formulation on the problem proposed in [3] composed of a source coil, a magnetic region and a conducting region. A reference solution is obtained with a T-ϕ Finite Element Method (FEM) at any frequency (from 0 to 1000 Hz). A mesh of 3,700 elements is used for volume integral **A-T** formulation, which allows obtaining a solution very closed to FEM (differences of less than 1.5% on the current losses, see Fig.2).

The **A-T** solution is strictly the same as that obtained by the **B-J** formulation proposed in [3]. We can notice that for the **B-J** formulation, the matrix of the final system to be solved is

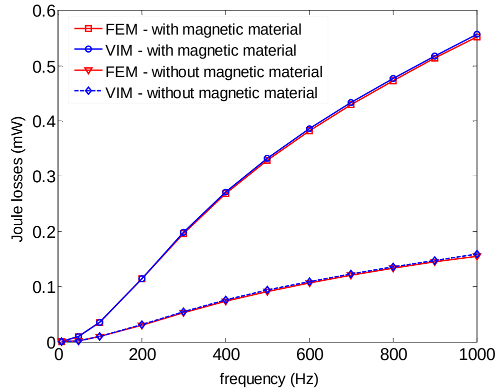

Fig. 2 Comparison of eddy current losses computed by FEM and VIM

obtained by replacing in (8) the matrices C_c and C_m by the independent loop matrices M_c and M_m, and is then very similar to **A-T** formulation. In practice, **A-T** formulation has the advantage of not requiring the determination of independent loops, which can be time consuming. Moreover, since the system is compatible, an iterative solver provides a unique solution without gauge condition [6] and a better convergence is observed with **A-T** formulation.

On the other hand **B-J** formulation allows to connect easily any external electrical circuit and to avoid any connectivity problem. In such cases, a coupling between the two approaches is then considered. It simply consists to uses mesh current (**J** formulation) instead of **T** in not simply connected conducting regions, and then determine independent loop matrix M_c instead of the use of connectivity matrix C_c.

V. Conclusion

Volume integral approaches combining **A** for magnetic region and **T** or **J** for electric regions are particularly well suited to model electromagnetic devices with large air domain. Moreover, in the case of simply connected conducting region, the formulation does not require the determination of independent loops and can be solved with an iterative solver without imposed a gauge condition.

VI. References

[1] P. Alotto, F. Desideri, F. Freschi, A. Machio, M. Repetto, “Dual-PEEC modeling of a two-port TEM cell for VHF applications,” IEEE Trans. Magn., vol.47, no.5, pp.1486-1489, Feb. 2011.

[2] A. Demenko, J.K. Sykulski "Geometric formulation of edge end nodal finite element equations in electromagnetics", The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, vol. 31, No. 5, 2012, pp. 1347-1357

[3] G. Meunier, O. Chadebec, Jean-Michel Guichon, "A Magnetic Flux – Electric Current Volume Integral Formulation based on Facet Elements for Solving Electromagnetic Problems", IEEE Transaction on Magnetics, Vol. 51, No. 3, March 2015.

[4] A. Bossavit, L. Kettunen, "Yee-Like Schemes on Staggered Cellular Grids: A Synthesis Between FIT and FEM Approaches", *IEEE Trans-Magn*, vol. 36, N°4, July 2000, pp 861-867

[5] R. Specogna and F. Trevisan, "Eddy-Currents Computation With T-Ω Discrete Geometric Formulation for an NDE Problem", IEEE trans-Magn, vol. 44, N°. 6, June 2008, pp 698-701

[6] Z. Ren, “The R.H.S. on the Convergence Behaviour,” IEEE Trans. Magn., vol. 32, no. 3, pp. 655-658, 1996